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I Semester M.Sc. Examination, February 2019
(CBCS Scheme)
MATHEMATICS
M107SC : Mathematical Analysis

Time : 3 Hours Max. Marks : 70

Instructions: 1) Answer any five questions.

2) All questions have equal marks.

1. a) Let (X, d_x) and (Y, d_y) be metric spaces, $E \subset X$ and $p \in E$. The function f is continuous at p, iff for every sequence $\{x_n\}$ in E converging to P, then show that the sequence $\{f(x_n)\}$ in Y converges to f (p).

   b) Show that continuous function on a compact metric space is uniformly continuous. (7+7)

2. Define connected metric space. Prove that a continuous mapping from a connected metric space into a metric space is connected. (14)

3. a) If f(x) is continuous on [a, b], f′(c) exists at some point $c \in [a, b]$, g(x) is defined on an interval I which contains the range of f(x) and g (x) is differentiable at the point f(c), if h(x) = g(f(x)), then show that h(x) is differentiable at c.

   b) With help of function $f(x) = x^2 \sin(1/x)$, prove the continuity of derivatives.(7+7)

4. a) State and prove L' Hospitals rule for indeterminate form of type ∞/∞.

b) If f (x) is real valued continuous function on [a, b] which is differentiable in (a, b) then show that there exist a point $x \in (a, b)$ such that $f(b) - f(a) = (b - a) f'(x)$. (7+7)

5. a) Every bounded sequence of real number contains a convergent subsequence. Prove or disprove it.

b) Prove that a sequence of real number converges iff it is Cauchy sequence. (7+7)

6. a) Show that e is irrational.

b) Prove : $\lim_{n \to \infty} \left(1+\frac{1}{n}\right)^n = e$. (7+7)

7. a) State and prove Cauchy integral test.

b) Given a series $\sum_{n=1}^{\infty} a_n$

Let $\alpha = \lim_{n \to \infty} \sup \sqrt[n]{|a_n|}$

then prove that

i) $\sum_{n=1}^{\infty} a_n$ converges if $\alpha < 1$

ii) $\sum_{n=1}^{\infty} a_n$ diverges if $\alpha > 1$

iii) if $\alpha = 1$, the test gives no information.

c) Test the convergence of

$\sum_{n=1}^{\infty} \left(\sqrt{n+1} - \sqrt{n-1}\right)$ (5+5+4)

8. a) State and prove Martin's theorem.

b) Suppose

i) $\sum_{n=0}^{\infty} a_n$ converges absolutely

ii) $\sum_{n=0}^{\infty} a_n = A$

iii) $\sum_{n=0}^{\infty} b_n = B$

iv) $C_n = \sum_{k=0}^{n} a_k b_{n-k}$, $n = 0, 1, 2 \ldots$

then show that

$$\sum_{n=0}^{\infty} C_n = AB$$

c) Assume that each $a_n \geq 0$, $n = 1, 2, \ldots$, then the product $\prod_{n=1}^{\infty} (1 + a_n)$ converges. Iff prove that the series $\sum_{n=1}^{\infty} a_n$ converges. (5+5+4)